Inversion methodology for determination of near-surface geology from seismic refraction amplitudes

Alan Meulenbroek
Velseis Pty Ltd and School of Earth Sciences, University of Queensland
PO Box 118, Sumner Park,
Queensland, Australia 4074
alanm@velseis.com

SUMMARY

The amplitude of a seismic refraction event is determined by the properties of rocks through which the seismic waves travel, the amplitude of the shot and the offset at which the refraction is recorded. A surface-consistent, non-linear inversion scheme, which uses the Levenberg-Marquardt algorithm, has been developed which aims to extract near-surface rock properties from the measured refraction amplitudes.

Perhaps the most important challenge to extracting a unique, geologically plausible solution is in determining initial values of control parameters which dictate how the solution is allowed to progress. If these control parameters are not tailored specifically to the problem in question, convergence to a solution can be very slow and even fail to get off the ground in some cases.

Comparison between results obtained using default control parameters, compared to those obtained using control parameters specifically tailored to the problem, shows a reduction in the error between the true observations and the model-generated observations, while retaining fidelity in the solution. A reformulation of the problem significantly reduces the error but fidelity is apparently compromised. Comments are made on how to achieve an optimal middle ground.

Key words: Inversion, least-squares, seismic refraction, amplitude.

INTRODUCTION

In a seismic survey using a surface source, the seismic velocity of the surface layer is generally unknown, unless a separate uphole or refraction survey is employed. This has implications in determining static time-shifts in the near-surface caused by variations in topography and weathering. Variations in layer velocities can also have an effect on the statics solution. Refraction travel-time data are routinely used for calculating the statics solution. However, without the knowledge of the surficial velocity, errors in estimated time shifts can cause errors in the critical near-surface depth model.

Often overlooked with refraction data is the amplitude of the refraction event. As with reflection seismology, the amplitude of a seismic event may provide additional information critical to the characterisation of the sub-surface. In the context of seismic refraction, the measured amplitude is proportional to the magnitude of the shot and the offset at which the refraction event is measured. The constant of proportionality is called the head-wave coefficient. This coefficient is a complex function of the elastic properties of the rock through which the critically refracted wave propagates (e.g. Červený and Ravindra, 1971).

Although many authors have derived and published theoretical expressions for the head-wave coefficient, very few have attempted to derive quantitative geological information from measured refraction amplitudes. In order to do this, the dominating amplitudes of the shot and offset terms must be separated from the head-wave coefficient term of interest. Palmer (2001a, 2001b) attempted to do this using a novel approach called the refraction convolution section (RCS). Meulenbroek (2010) presented an alternative technique in which a surface-consistent, non-linear least-squares inversion scheme in employed. The formulation of this inverse problem has similarities to that of surface-consistent residual statics.

This paper extends on Meulenbroek (2010), focussing more on the inversion methodology, namely the means of forcing the problem to a realistic solution as fast as, and with as little error, as possible.

THE INVERSE PROBLEM

With reference to Figure 1, Equation 1 shows the expression for the refraction amplitude originating from a source, *S*, with a magnitude, $F(t)$, recorded by a geophone, *G*, which is at an offset, *r* from *S*.

$$Amplitude = \frac{KF(t)}{(rL^3)^{1/2}} \qquad (1)$$

The distance the wave travels in the refractor, *L*, depends on the depth to the refractor, *z*, and the critical angle of refraction, i_C.

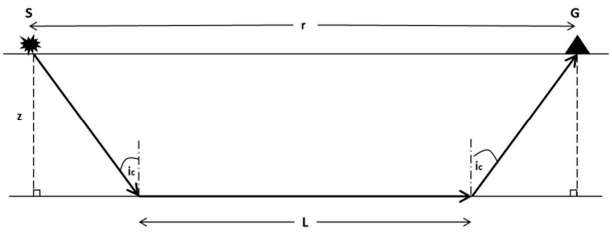

Figure 1: Ray-path diagram of critically refracted wave originating from the source, *S*, and recorded by a geophone, *G*, at an offset, *r*.

Each individual refraction amplitude measurement originates from a unique combination of model parameters. These model parameters are the shot term ($F(t)$), the receiver term (K) and the offset term (denominator in Equation 1). This can be expressed as a system of equations of the form:

$$\boldsymbol{Ax} = \boldsymbol{b} \qquad (2)$$

Here **x** is the N vector of model parameters; **b** is the M vector of observations and **A** is the MxN matrix relating each observation to the model parameters. Because M>N, the system is overdetermined and can be solved using the least-squares criterion. However, because the observations are a non-linear combination of model parameters (Equation 1), the system is non-linear and cannot be solved using simple linear least-squares inversion techniques (such as those used for residual statics). The nonlinear problem is given by:

$$b_i = \prod_{j=1}^{n} \left(A_{ij} x_j \right) + e \qquad (i = 1, m) \tag{3}$$

where e is the error term. The algorithm used to solve this nonlinear problem is the Levenberg-Marquardt algorithm (Levenberg, 1944; Marquardt, 1963). For this study, a widely used implementation of the algorithm, namely the PEST suite of programs (Doherty, 2004, 2010), is used. In the following, the PEST terminology is broadly used, although the comments are applicable to non-linear inversion algorithms in general.

Meulenbroek (2010) presented such an inversion for a 2D dynamite survey in the Bowen Basin. The resulting model parameters were realistic and consistent with Palmer's method. However when they were back-substituted, the error between the true observations and the model-generated observations was very high. The objective function, ϕ, which is the sum of the squares of these residual errors, is an important indicator of the inversion performance. The aim is to reduce ϕ as much as possible while still producing a realistic result. Informed decisions about the inversion control parameters, as well as how the problem is formulated, can have a substantial effect on how well the problem is optimised, if at all.

PRACTICAL CONSIDERATIONS

Practical non-linear inversion algorithms include various control parameters which, when tuned correctly to the specific problem, can mean the difference between an effective and an ineffective inversion. These important control parameters include initial value of the Marquardt-λ, and how much this parameter is allowed to change as optimisation progresses. A high initial λ will take advantage of the properties of the steepest descent method while a low initial λ will take advantage of the properties of the Gauss-Newton method (e.g. Doherty, 2010).

Regularisation, i.e. the application of a-priori information for the purposes of constraining the model parameters, is a key step in turning an ill-posed problem into a well-posed problem. Regularisation in this particular problem takes the form of eliminating amplitudes which are either anomalously large at a given offset, or have been clipped at near offsets. It is also possible to limit the upper and lower bounds of parameter values so that they are forced to remain within a realistic range. It should be noted that because the problem is overdetermined, the solution will never be exact.

Weighting the observations can also have a positive effect on the outcome of the inversion. In the case of raw refraction amplitude observations, there can be up to 5 orders of magnitude difference between the amplitudes measured at the near-traces and those measured at far traces. In this case, the quantity described by the larger numbers will dominate ϕ, thus inhibiting the optimisation process (Doherty, 2010). Applying weights to the data can overcome this problem by reducing the large residuals, hence making the other model parameters more visible to the calculation of ϕ.

Instructions regarding when to cease optimisation are just as important as those regarding how to start optimisation. If these control parameters are not chosen judiciously, optimisation may cease prematurely, producing a poorer result than could otherwise be achieved. Conversely, if these are set too tight, optimisation may not cease at all. These control parameters include the maximum number of iterations, the relative ϕ reduction and a lower ϕ threshold where the user can deem the optimisation to be complete.

Instructing the problem as to how the model parameters are allowed to change over time is another critical control which can have a major effect on how quickly a problem can be optimised. If the initial model-generated observations are vastly different to the true observations, a small limit on how much each parameter is allowed to change per iteration can severely restrict optimisation, making progress extremely slow. Limiting how much ϕ is allowed to change per iteration can also have this effect.

Each of these particular control parameters, and many others, must be specifically chosen to suit the model in question.

Other Tools

The use of truncated singular-value decomposition (TSVD) in the problem at the stage of calculating the parameter upgrade vector (Doherty, 2010: Equation 2.18) can help eliminate unwanted noise in the problem. When inverted, this random noise is amplified and then dominates the solution, making it unstable. If the matrix has any singular values of zero, the matrix is singular and cannot be inverted. By eliminating the eigenvectors representing certain parameter combinations which cause this problem, identified by their small or zero singular values, the matrix to be inverted becomes less singular or non-singular. The advantage of this is that the problem may be forced to a solution. However, the disadvantage is that it can also eliminate fine detail in the solution. If optimisation is terminated due to the matrix being singular, application of this may allow the optimisation to get off the ground.

In PEST, Broyden's Jacobian matrix update is also used as standard. In short, this technique aims to reduce the computational expense of calculating the Jacobian matrix at each iteration. The reader is referred to Broyden (1965) and Doherty (2010) for further information.

REAL DATA EXAMPLE

Figure 2 shows the result for the inverted shot, receiver and offset terms using a simple initial model and default control parameters generated by PEST (Meulenbroek, 2010). The runtime for this solution was 5 hours. In that time, there were 2 iterations. The final ϕ was large (7.21×10^{21}) with only minimal reduction on the second iteration.

Despite the apparently high ϕ value, analysis of the constituent shot, receiver and offset domains did yield apparently meaningful information. The relative amplitudes of the shots seen in Figure 2 were broadly consistent with the relative amplitudes seen on the raw shot records. In the offset domain, the amplitudes exhibit a general inverse square relationship. In the receiver domain, a smoothed version of the inverted amplitudes were consistent, in a relative sense, with those obtained using the RCS method of Palmer (2001a, 2001b). Nevertheless the error indicator, ϕ was still unsettlingly high. Tuning the control parameters specifically to the problem may help to reduce ϕ.

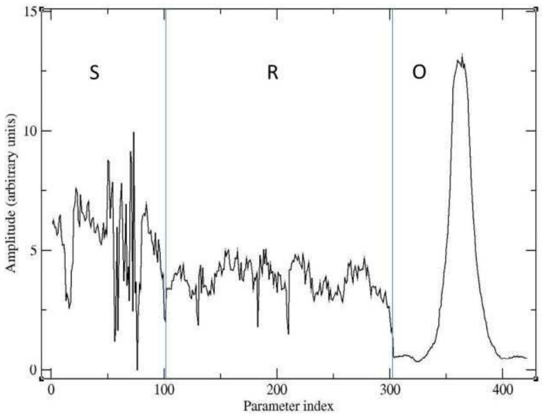

Figure 2: Amplitude due to shot, receiver and offset terms obtained from inversion of refraction amplitude data using default control parameters.

Using the default control parameters, optimisation was terminated because ϕ was not being reduced at all. The maximum relative parameter change was set to 3.0 which, considering the large observational range, is very restrictive to how the model parameters could move. For each iteration, the parameters representing the near-offset amplitudes moved the most, which is to be expected because their final values differ the most from their initial values. This suggests that the limit of how much each parameter can change per iteration should be much greater.

As a comparison, the following control parameters were changed from default values to model specific values:

- Maximum parameter value change limit changed from 3 to 30000,
- TSVD activated,
- Termination criteria tightened so relative changes in ϕ are very small and tested over a large range of λ,
- λ adjustment parameter set to allow it to change as much as possible,
- Relative ϕ reduction limit made very small ($3x10^{-4}$) to allow more λ values to be tested per iteration. This reduces the number of required iterations.
- Absolute/relative parameter increment for shot and offset terms increased to 1000 so they are able to move more freely,

Figure 3 shows the resulting amplitudes for the shot, receiver and offset domains, separated by vertical lines. This result took 8 hours to evaluate 8 iterations, in which time ϕ was reduced from $7.29x10^{21}$ to $5.48x10^{20}$.

Comparison between the vertical scales of Figure 2 and Figure 3 shows that there is a large difference between the constituent shot and offset amplitudes. When the problem specific control parameters are used, the result is a lower ϕ. Although ϕ is still very high, the fact that it is lower suggests that the result shown in Figure 3 is closer to the truth than the result shown in Figure 2. The result in Figure 3 illustrates the relative amplitude of the receiver terms compared to the dominant shot and offset terms. These terms in Figure 3 are very similar in character, to those in Figure 2, although the relative scaling of the shot, receiver and offset terms is very different.

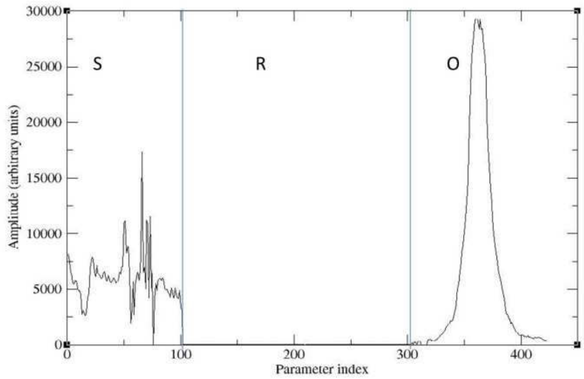

Figure 3: Amplitude due to shot, receiver and offset terms obtained from inversion of refraction amplitude data using control parameters tailored specifically to the problem.

Figure 4 shows a close-up of the receiver terms in Figure 3. Comparison with Figure 2 suggests that the general trend of the head-wave coefficient has been preserved. However, the original inversion result appears to have a higher resolution, i.e. the fine detail appears to be better preserved than the result shown in Figure 4. This may be attributed to the application of TSVD. In this case, stability has been added, but at a slight cost of resolution.

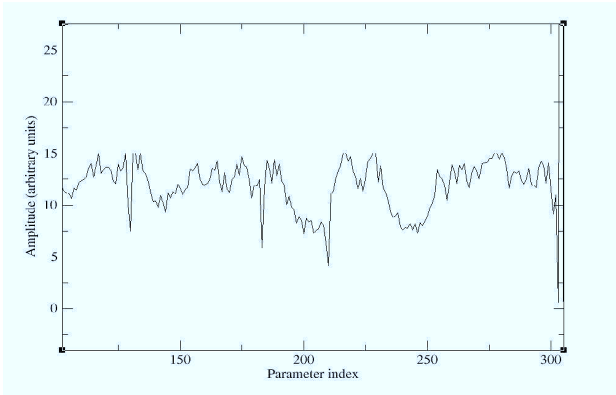

Figure 4: Zoom in of receiver terms from Figure 3.

Although ϕ has been reduced by 1 order of magnitude, it is still very high. The reason for this is the very large range of observational amplitudes. The problem is formulated such that large residuals dominate ϕ, hiding the rest from contributing to the solution (Doherty, 2011 pers. comm.). This can be overcome in several ways. One way is to weight the data such that every observation contributes to the calculation of ϕ equally. This can be done either by applying a weight to each observation which is the inverse of its value or to weight the data based on how many observations per offset there are. Another solution entirely is to reformulate the problem by attempting to eliminate the large residuals prior to inversion.

Eliminating Large Residuals

If the observed refraction amplitudes are sorted based on their offsets and plotted, a clear relationship can be seen (Figure 5). As is stated above, the amplitudes measured at near offsets differ from those measured at far offsets by about 5 orders of magnitude.

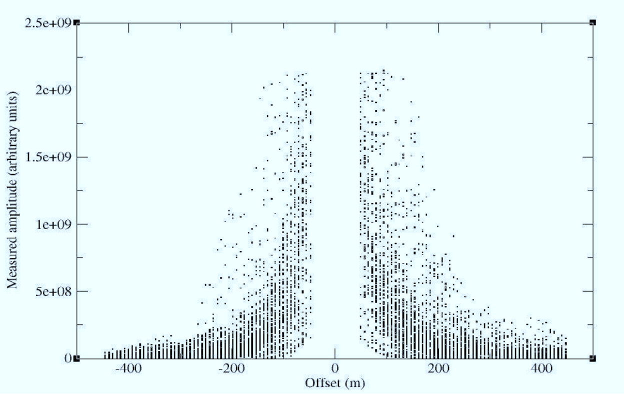

Figure 5: Measured refraction amplitude plotted against offset (m).

To mitigate this sharp decrease in amplitude with offset, the mean amplitude at each offset is calculated and an inverse power law curve is fitted to these mean data points. The data are then multiplied by the inverse of this curve to equalise the amplitudes. The positive and negative offsets are treated separately. These are subsequently input as the true observations.

Based on the initial result shown in Figure 2, the initial model was chosen with all parameters set to 4.0. The initial ϕ for this model is 2.96×10^7. After 11 iterations, ϕ is reduced to 2874.9. Figure 6 shows the result. The vertical lines separate the shot, receiver and offset domains.

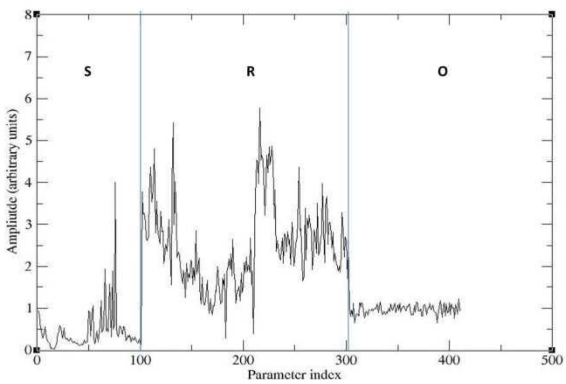

Figure 6: Amplitude due to shot, receiver and offset terms obtained from inversion of refraction amplitude data after offset dependence has been removed prior to inversion.

Note how the dominating offset effect seen in Figures 2 and 3 has been all but eliminated. There is only random residual noise in the offset domain. Although ϕ has been reduced significantly, comparison between Figure 6 and Figures 2 and 4 suggests that this has come at the expense of geological certainty in the receiver domain. Because Meulenbroek (2010) showed that the receiver terms in Figure 2 are relatively consistent with the results obtained using Palmer's method, the fact that the receiver terms in Figure 6 differ suggests that while targeting the offset terms to reduce the residual error, the shot and offset terms have also been affected. Clearly a compromise must be reached whereby ϕ is reduced if possible, but with care taken to ensure the geological fidelity of the solution.

CONCLUSIONS

This study has examined non-linear inversion methodology aimed at extracting head-wave coefficients (and ultimately geological interpretation) from refraction amplitudes. This demonstrates that it is important to consider the specific problem in question when designing an inverse problem. Setting the control parameters, which dictate how the problem is set up, how the optimisation is allowed to progress and how it is terminated, requires careful consideration if the inversion is to be both meaningful geologically and mathematically robust.

Initial results demonstrate that the fit between the model-generated observations and the true observations (as indicated by the objective function ϕ) can be improved if inversion control parameters are tailored specifically to the problem at hand. However, it is unwise to rely on any single criterion to automatically judge the success, or otherwise, of the inversion. The geological fidelity of the derived solution must be carefully monitored. Attempting to define a happy medium between mathematical optimisation and the geological reality is the subject of current research.

ACKNOWLEDGMENTS

The real data presented in this paper have been made available with the support of the Australian Coal Association Research Program (ACARP). The author also thanks Dr. John Doherty, the author of PEST, for his insight and ongoing consultation. This research has been, in part, supported by to the ASEG research foundation.

REFERENCES

Broyden, C.G., 1965, A class of methods for solving nonlinear simultaneous equations, Mathematics of Computation, **19**, pp577-593.

Červený, V., Ravindra, R., 1971, Theory of seismic head waves, Toronto, University of Toronto Press.

Doherty, J., 2004, PEST – Model-Independent Parameter Estimation User Manual: 5th edition, Watermark Numerical Computing.

Doherty, J., 2010, Addendum to the PEST Manual, Watermark Numerical Computing.

Levenberg, K., 1944, A method for the solution of certain nonlinear problems in least squares, Quarterly of Applied Mathematics, **2**, pp164-168.

Marquardt, D.W., 1963, An algorithm for least squares estimation on non-linear parameters, Journal of the Society of Industrial and Applied Mathematics, **11**, pp431-441.

Meulenbroek, A., 2010, Least-squares inversion of refraction amplitudes for near-surface velocity control, Extended Abstracts, ASEG 21st International Conference and Exhibition, Sydney.

Palmer, D., 2001a, Imaging refractors with the convolution section, Geophysics, **66**, 5, 1582-1589.

Palmer, D., 2001b, Resolving refractor ambiguities with amplitudes, Geophysics, **66**, 5, 1590-1593.